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Pressure drop correlation for laminar, mixed convection, aiding flow heat transfer in a vertical tube

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Abstract

Pressure drop is significantly affected by heat transfer in mixed convection situations. A semi-empirical correlation for pressure drop is developed from a theoretical base by first making use of the momentum integral solution to the heat and momentum equations for natural convection on a vertical surface. The boundary condition on the free-stream side of the boundary layer is changed to reflect the shear on that surface due to the forced convection, and empirical data are used to develop a formula useful in design and applications. The equation may be used with bulk fluid properties or film properties. The equation is valid for laminar, mixed convection conditions in vertical, internal, aiding flows with constant wall temperature boundary condition.

Keywords: Mixed convection heat transfer; Vertical tube; Aiding flow; Pressure drop

1. Introduction

Mixed convection heat transfer exists when natural convection currents are the same order of magnitude as forced flow velocities. The term "combined convection" is also used, and the flows may be internal or external to a bounding surface. Forced flows may be horizontal, vertical or some angle in between. In vertical, internal flows the buoyancy forces may be directed opposite to the forced flow (hence "opposing flow" terminology), or in the same direction as the forced flow (hence the "aiding flow" terminology). In opposing flow situations (heating in downflow or cooling in upflow) the velocities in opposite directions create shear instability and turbulence. This can happen even when the Reynolds number based on forced flow average velocity is in the laminar region. In aiding flows (heating in upflow or cooling in downflow), by contrast, the velocities of forced flow and buoyancy forces are in the same direction, and laminar-like flow is preserved even if the Reynolds number based on forced flow average velocity is nominally in the turbulent region. Hence aiding flow situations are amenable to laminar flow analysis, and that is the subject of the present work.

The interactions of buoyancy-driven components of velocity and forced flow velocities can have a profound effect on the velocity profile, heat transfer coefficient and pressure drop. The literature is wide and varied. A review by Jackson et al. (1989) is particularly helpful. Early work of Scheele and Hanratty (1963) and Zeldin and Schmidt (1972) further elucidate aspects of aiding flow, in particular.

The measurement of pressure drop in vertical, mixed convection flow is a very difficult problem, particularly for liquids, because the actual pressure differences are quite small. Measurement of liquid pressure drop is almost impossible to achieve with anything but a manometer. Manometer readings need to be corrected because the fluid inside the conduit is at a different temperature than that outside the conduit, and a correction factor due to the difference in density must be applied. This correction factor depends on flow rate and must be arrived at by numerical integration of radial and axial velocity profiles, which must also be obtained. This creates a lot of difficulty for experimental investigations with liquids, in particular. The work of Martinelli et al. (1942) was used by Saylor and Joye (1991) to arrive at a correction procedure. We further extend that

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Nomenclature

C_p	fluid heat capacity (kJ/kg K)
C_1, C_2, C_3	constants defined in the text
D	tube diameter (m)
f	Fanning friction factor (dimensionless)
g	gravitational acceleration (m/s^2)
Gr_L	Grashof number based on tube length, $\rho^2 L^3 g\beta\Delta T/\mu^2$
h	film heat transfer coefficient (W/m^2 K)
k	fluid thermal conductivity (W/m K)
L	heated length of tube (m)
P	pressure (Pa or mm water)
ΔP	pressure drop (Pa or mm water at max density)
Pr	Prandtl number, μC_p/k
Q	volumetric flow rate (m^3/s)
r	radial coordinate (m)
R	tube radius (m)
Re	Reynolds number, $\mathrm{D}v\rho/\mu$
Sp.Gr.	specific gravity—density relative to that of water at 4 °C
T	temperature (K)
ΔT	temperature difference (K)
u, v	fluid velocity (m/s)
y	distance coordinate perpendicular to the wall (radial direction) (m)
z	tube axial distance coordinate (m)

Greeks

β	volume expansion coefficient of the fluid (1/K)
$\dot{\gamma}$	shear rate, $-\mathrm{d}v/\mathrm{d}r$ (s^{-1})
δ	boundary layer thickness (m)
φ_v	viscosity ratio, μ/μ_w (dimensionless)
μ	fluid viscosity (mPa s)
v	kinematic viscosity, μ/ρ (m^2/s)
ρ	fluid density (kg/m^3)
τ	shear stress (Pa)

Subscripts

Av	average value
b	properties of fluid at average bulk temperature
D	based on diameter
expt	experimental values
f	properties of fluid based on average film temperature, $(T_w + T_b)/2$
L	based on length
lam	laminar flow
nc	natural convection
w	evaluated at the wall conditions
∞	relative to free stream conditions

work here to develop a predictive equation for pressure drop in vertical, internal, aiding flow situations with constant wall temperature.

2. Momentum integral approach

The expected increase in pressure drop (or friction factor) in aiding flow may be calculated for situations where the velocity profile is known. For mixed convection with constant wall temperature situations, the theory is not well developed, and an alternative approach may be used. The momentum integral analysis for velocity profile in a purely natural convection boundary layer in vertical flow can provide a useful starting point for development. For convenience, a mixed convection flow may be divided into a "core" region in the center of the conduit—a tube, for example—and a "shell" region near the wall. The core region is where forced flow dominates. The shell region is assumed to be where natural convection dominates; it is essentially the natural convection boundary layer. The velocity in this boundary layer is a function of radial and vertical distance coordinates. Holman (1997) illustrates the momentum integral approach and gives,

$$u/u_z = (y/\delta)(1 - y/\delta)^2 \tag{1}$$

where y is taken as the radial coordinate for the present situation of flow in a tube, δ is the boundary layer thickness, u is the velocity and u_z is a velocity function given by

$$u_z = C_1 z^{1/2} \tag{2}$$

where z is the vertical distance coordinate and C_1 is a constant given by

$$C_1 = 5.17v(0.952 + Pr)^{-1/2}(g\beta(T_w - T_b)/v^2)^{1/2} \tag{3}$$

where Pr is the Prandtl number, β is the volume expansivity, T_w is the average wall temperature and T_b is the average bulk temperature of the fluid, v is the kinematic viscosity (μ/ρ), and g is the acceleration of gravity.

The boundary layer thickness, δ, is also a function of z. Thus,

$$\delta = C_2 z^{1/4} \tag{4}$$

where C_2 is another constant given by

$$C_2 = 3.93(0.952 + Pr)^{1/4}(g\beta(T_w - T_b)/v^2)^{-1/4}Pr^{-1/2} \tag{5}$$

This development, attributed to an unpublished work of Squire, first appeared in Modern Developments in Fluid Dynamics (Goldstein, 1938), and was modified by Eckert and Drake (1950) before appearing in its most recent form (Holman, 1997).

The solution depends on four boundary conditions:

(1) $u = 0$ at $y = 0$ (the wall),
(2) $u = 0$ at $y = \delta$ (the boundary layer thickness),
(3) $\partial u/\partial y = 0$ at $y = \delta$, and
(4) $\partial^2 u/\partial y^2 = -g\beta(T_w - T_b)/v^2$ at $y = 0$,

where T_b has been substituted for T_∞. In the mixed convection case, the second boundary condition would not hold; the velocity at $y = \delta$ is non-zero due to the forced flow. This will be addressed later.

The average boundary layer thickness, δ_{av}, and the average velocity, u_{av}, can be obtained by integrating Eqs. (2) and (4), respectively. This yields,

$$\delta_{av} = 0.8\delta|_{z=L} \tag{6}$$

and

$$u_{z,av} = 0.667u_z|_{z=L} \tag{7}$$

Values for u_z and δ at the top of the tube can be obtained by setting $z = L$ in Eqs. (2) and (4). The average velocity must be calculated from a double average—with respect to both radial and axial coordinates. The radial coordinate does not depend on the thermal properties or experimental conditions, and so integrating Eq. (1) from $y/\delta = 0$ to $y/\delta = 1$ gives,

$$u_{av} = 0.0833u_{z,av} \tag{8}$$

Using the tube dimensions and conditions of the data of Saylor and Joye (1991), $L = 1.57$ m, $L/D = 49.6$, $Pr = 4.53$, $Gr_L = 1.22 \times 10^{13}$ (this is the Grashof number based on length). Thus,

$$C_1 = 2.69 \tag{9}$$

$$u_{z,av} = 2.25 \text{ m/s} \tag{10}$$

and

$$u_{av} = 18.7 \text{ cm/s} \tag{11}$$

$$C_2 = 0.00212 \tag{12}$$

Thus,

$$\delta_{av} = 1.9 \text{ mm} \tag{13}$$

By contrast the tube ID is 32 mm. This gives a ratio of 1/20 for the boundary layer thickness to diameter. This is a large enough ratio to ignore the effects of curvature on the boundary layer and to permit the analysis of external flow to fit an internal flow situation. If δ were the same order of magnitude as D, the tube diameter, this could not be said.

An estimate of the friction factor can be made for upflow heating or downflow cooling cases, because aiding flow situations stabilize laminar flow, and the equations developed from the momentum integral approach will approximate these conditions reasonably well. However, in opposing flow situations, instability is inherent. Turbulence will always be present, and the momentum integral analysis using laminar relations will not apply.

In upflow heating or downflow cooling, these equations can be used along with the defining equations for Fanning friction factor, f, defined below,

$$f = 2\tau_w/\rho v_{av}^2 \tag{14}$$

and Newton's Law for viscosity,

$$\tau_w = -\mu(dv/dr)_w \tag{15}$$

where ρ is the fluid density, μ is the viscosity, v is the fluid velocity, and τ_w is the shear stress at the wall. At a fixed flow rate, the friction factor for purely forced convection in laminar flow would be $16/Re$, where Re is the Reynolds number. The shear stress at the wall can be estimated if the velocity profile is known, and a comparison to the purely forced flow case can be made. The film temperature is different than the bulk average temperature, so viscosity in the boundary layer "shell" region is different than that in the "core" region filling the rest of the tube space. The shear rate at the wall can be estimated by taking the derivative of Eq. (1) using the average thickness for δ and letting $y = R - r$.

$$-dv/dr|_w = u_{z,av}\, d((y/\delta)(1 - y/\delta)^2)_{y=0}/dy \tag{16}$$

which yields $u_{z,av}/\delta$. Thus, the shear rate at the wall based on the velocity profile approximation is about 1000 s^{-1} for the data of Saylor and Joye (1991) quoted above.

The shear rate expected for laminar forced flow alone at the same conditions depends only on Reynolds number. At $Re = 360$, the flow rate is 3.8 cm^3/s, and the shear rate can be calculated from a standard tube-flow equation, given for example by Rosen (1982) as

$$-dv/dr|_w = 4Q/\pi R^3 = (4)(3.8)/\pi(1.6)^3 = 1.0 \text{ s}^{-1} \tag{17}$$

where Q is volumetric flow rate, and R is tube radius. The shear rate and shear stress are directly proportional to Q in laminar flow. At $Re = 2820$ (one of the data points taken from Fig. 3 of Saylor and Joye (1991)), the shear rate would be

$$-dv/dr|_w = 7.8 \text{ s}^{-1} \tag{18}$$

The shear stress, friction factor and pressure drop are all proportional to the shear rate. Therefore, in upflow heating and other aiding flow situations in the low Reynolds number, laminar region, the friction factor can be *two-to-three orders of magnitude higher* than that based on forced flow considerations alone. This is quite surprising and is shown by the data of Saylor and Joye (1991), for example, Fig. 3 in that reference, and this substantially explains the increased pressure drops reported there for aiding flow. The corresponding heat transfer coefficients are reported in Joye et al. (1990).

3. Development of predictive theory

A more quantitative relationship that could be used to predict the pressure drop in this kind of flow with heat transfer would be desirable. Since the foregoing laminar analysis would seem to be valid, the pressure drop increase due to aiding flow heat transfer could be calculated on the basis of shear rates, as indicated above. The predictive equation would then take the general form,

$$\Delta P/\Delta P_{\mathrm{lam}} = 1 + \dot{\gamma}_{\mathrm{nc}}/\dot{\gamma}_{\mathrm{lam}} \tag{19}$$

where subscript "lam" refers to the laminar forced flow in the absence of natural convection effects, and the subscript "nc" refers to the shear rate including the effect of the natural convection forces, and $\dot{\gamma}$ is the shear rate. The shear rates can be computed by

$$\dot{\gamma}_{\mathrm{nc}} = u_{z,\mathrm{av}}/\delta_{\mathrm{av}} = 0.834L^{1/4}C_1/C_2 \tag{20}$$

$$\dot{\gamma}_{\mathrm{lam}} = 4Q/\pi R^3 = 8\mu Re/\rho D^2 \tag{21}$$

and

$$\dot{\gamma}_{\mathrm{nc}}/\dot{\gamma}_{\mathrm{lam}} = 0.137\nu_{\mathrm{f}}Gr_{\mathrm{L}}^{3/4}Pr^{1/2}/\nu_{\mathrm{b}}Re(L/D)^2(0.952+Pr)^{3/4} \tag{22}$$

The quantity ΔP_{lam} can be computed by

$$\Delta P_{\mathrm{lam}} = (128\mu LQ/\pi D^4)(\rho g)(\mathrm{Sp.Gr.}/\phi_{\mathrm{v}}^{0.38}) \tag{23}$$

which is the well known Hagen–Poiseuille equation for laminar flow in a tube, modified for units of pressure in mm water at maximum density and corrected for heat transfer (Holman, 1997).

Eq. (22) needs correction for mixed convection conditions. The second boundary condition for the momentum integral equations must be changed. The velocity is finite and unknown at the boundary layer edge. Using this boundary condition in this form results in an analytically unsolvable set of equations. In searching for another route to a solution of this problem, one can see that this boundary condition clearly introduces an additional Reynolds number dependence, because the velocity at the boundary will depend on the forced flow. It is possible that an additional Grashof and Prandtl number dependence could also be introduced. The Reynolds number dependence can be recovered from the pressure drop data of Saylor and Joye (1991) since the Grashof number was essentially constant and Prandtl number variation was small.

Therefore, an equation of the form below can be postulated, rearranged, and plotted against Reynolds number to evaluate the power "n" and the constant "C_3". In this way all dependencies can be recovered so long as Gr and Pr do not depart too significantly from those of the data. The actual measured pressure drop is used for ΔP, and ΔP_{lam} is calculated by Eq. (23),

$$\Delta P/\Delta P_{\mathrm{lam}} = 1 + C_3(\nu_{\mathrm{f}}/\nu_{\mathrm{b}})Gr_{\mathrm{L}}^{3/4}Pr^{1/2}/(0.952+Pr)^{3/4} \times (L/D)^2Re^n \tag{24}$$

Fig. 1 shows the pressure drop data from Saylor and Joye (1991) rearranged in a format suitable to determine the slope and the constant C_3. This requires moving the "1" to the left-hand side of the equation. From Fig. 1 the slope is evaluated as −2 and the constant $C_3 = 76.3$.

In Eq. (24) the film properties were used to evaluate both the Grashof and Prandtl numbers as would be logical from considerations of the boundary layer. The bulk properties were used for Reynolds number, because it governs the forced flow in the "core" more than the boundary layer in the "shell". In many instances it may be preferred to use the bulk properties for all dimensionless groups to put them on the same basis. Thus Eq. (24) can be re-evaluated using the bulk properties. The data need to be re-plotted, and the corresponding line is also shown in Fig. 1. The corresponding equation is given below,

$$\Delta P/\Delta P_{\mathrm{lam}} = 1 + 1565Gr_{\mathrm{L}}^{3/4}Pr^{1/2}/(0.952+Pr)^{3/4}Re^2 \times (L/D)^2|_{\mathrm{b}} \tag{25}$$

where all fluid properties are evaluated at the bulk average temperature. This equation is much simpler than the complex Bessel function relationships developed originally by Hallman (1956) which lack a clear Reynolds number dependence. The pressure drop ratio

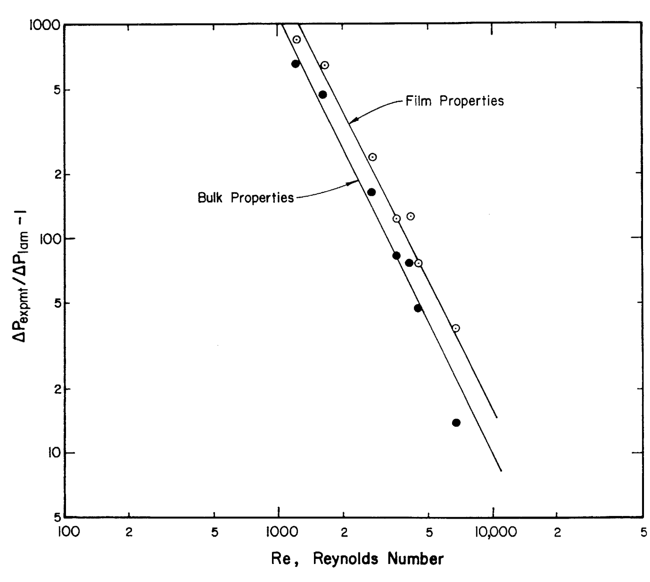

Fig. 1. Slope and intercept determination for Reynolds number dependence.

term, $\Delta P/\Delta P_{\mathrm{lam}}$ is identical to $fRe/(fRe)_0$, used by others in this field to characterize pressure drop functionality.

In using this equation one must know something about the temperatures of the fluid. In general this is not possible a priori. However, the bulk average temperature of the fluid could be reasonably estimated by taking an average of the wall temperature and the entrance temperature, both of which should be known or, in the case of the wall temperature, reasonably estimated. This should be a very close estimate for property determination and Grashof number calculation. A second iteration may be required after the system temperatures are better defined. In many cases the diameter-based Grashof number may be preferred. This is easily found by substituting "D" for "L" in the definition and calculating the conversion factor to use in Eq. (25), i.e.

$$Gr_{\mathrm{L}} = (L/D)^3 Gr_{\mathrm{D}}$$

4. Comparison with data

Fig. 2 shows how well this equation fits actual pressure drop data from Saylor and Joye (1991) at $Gr_{\mathrm{L}} = 4 \times 10^{12}$ and $Pr = 4$. The fit is quite good despite the fact that Grashof number based on bulk average temperature instead of film temperature is not quite so constant (it varies almost an order of magnitude from highest at the low flow rates to lowest at the high flow rate). The corresponding forced flow Reynolds numbers are indicated for each data point. The theoretical pressure drop calculated by the Hagen–Poiseuille equation for laminar flow and the pressure drop calculated using the friction factor equation for turbulent flow are also given to show the relative order of magnitudes between the two. Fig. 2 also shows the difference between the pressure drop calculated purely on the basis of forced flow considerations without the influence of buoyancy forces and the pressure drop calculated with significant buoyancy forces acting. From the graph, one can see the differences between these pressure drops are quite substantial, yet the fit to the actual data provided by Eq. (25) is quite good. The plot is pressure drop vs. flow rate, as normally would be used. One can see clearly that without a heat transfer correction, the pressure drops calculated on the basis of forced flow alone, laminar or turbulent, will be in significant error.

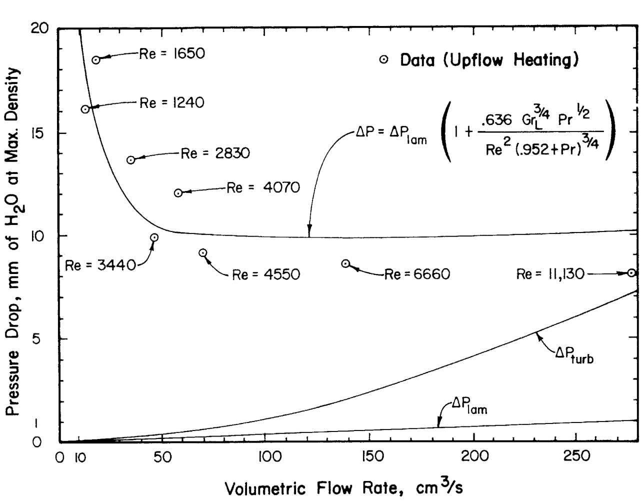

Fig. 2. Prediction of theory compared to data at $Gr_{\mathrm{L}} = 4 \times 10^{12}$ and $Pr = 4$.

The range of applicability of Eq. (25) is in the laminar, mixed convection zone, which is defined by the heat transfer and not the usual Reynolds number of 2100 as in forced flow alone. One can see from Fig. 2 that laminar-like flow extends into the turbulent forced flow region up to Reynolds numbers of about 7000 at this Grashof number. At Reynolds number of 11,000 the pressure drop measured is almost equal to that calculated by turbulent forced flow, and hence the significant effect of buoyancy disappears around this value. Up unto this value the flow characteristic is predominantly laminar. Similar things happen to the heat transfer coefficient, viz. the Nusselt values approach those for purely forced convection at around the same Reynolds number for this Grashof number (Joye et al., 1990). The transition is dependent on Grashof number as illustrated by Joye (1996) and Joye and Wojnovich (1996). The equation is expected to be valid for Grashof numbers in the range up to about 10^8 based on diameter. This corresponds roughly to a temperature difference between wall and bulk fluid of up to about 60 °C for aqueous liquids. This range is typical for heat transfer practice in heat exchangers.

One extremely important point for experimental data collection is that a calming section, or unheated entrance length, for developing the hydrodynamic velocity profile prior to the heated section of the tube is absolutely essential for correct measurement of pressure drop. The L/D of this calming section was 30 for the experiments of Saylor and Joye (1991), and this was found to be sufficient. In subsequent experiments, we have found that dispensing with the hydrodynamic entrance length gives completely erroneous results in pressure drop, and L/D of about 20–25 would be the absolute minimum required for a good pressure drop determination. Measuring small pressure drops in liquid flows is quite challenging and requires the utmost care in the design and operation of the experimental apparatus. Estimated error in the measured pressure drop is about ±15% at the low Reynolds numbers and about ±8% at the higher Reynolds numbers.

5. Comparison with numerical results

There have been a host of numerical studies on internal mixed convection flows in the recent past. Almost

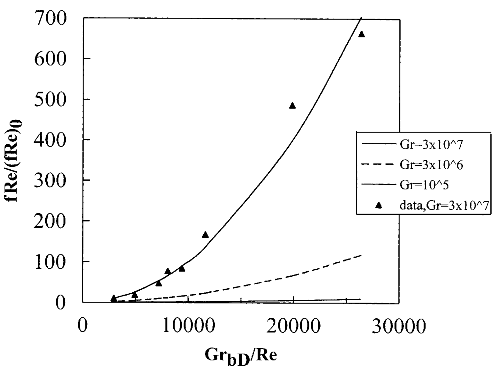

Fig. 3. Pressure ratio of the present theory plotted against Gr/Re with Gr as parameter.

none deal with the present situation, most using the constant heat flux boundary condition and a geometric cross section other than circular. Busedra and Soliman (1999) give a good summary of recent work and include their own for inclined channels of semicircular cross-section. They present the pressure drop results of their numerical studies in terms of $fRe/(fRe)_0$ vs. Gr/Re. Fig. 3 shows the result of the present theory in that format. The y-axis is the exact equivalent of the pressure drop ratio of Eqs. (24) and (25). The shapes of the curves between this work and that of Busedra and Soliman are quite similar, but the pressure drop ratios are much higher in the present work. Busedra and Soliman acknowledge that pressure drop ratios can be higher in circular tubes and show some data that indicate it could be 50–100% higher than the values they obtain for ducts of semi-circular cross section. However, the results in Fig. 3 are much higher than that. The reasons have to do with the vastly increased shear rate at the wall resulting from the buoyancy currents increasing the velocity in the boundary layer near the wall. Hallman shows the possibility of 100-fold increase in pressure drop for Rayleigh numbers ($GrPr$) of 10,000 for constant wall temperature situations. Velusamy and Garg (1996) show numerical results for vertical tubes with constant heat flux boundary condition and give a pressure drop ratio of about 15–20 at Rayleigh numbers of about 10,000. This is quite a bit higher than Busedra and Soliman and suggests that results for constant wall temperature boundary condition will not be the same as those for constant heat flux boundary condition, and the geometry plays a very significant role. Indeed, Busedra and Soliman (1999) show that different results are obtained from different boundary condition, though their curves show only minor differences. This could be attributed to the fact that they used 3.66 as the limiting Nusselt number for constant wall temperature situations, but it is much more complicated than that (see Martinelli et al., 1942).

Fig. 3 also shows a very significant effect of Grashof number on the pressure drop ratio. For Grashof numbers (based on diameter and bulk fluid properties) up to about 10^5, there is not much effect of buoyancy on pressure drop. For liquids, this corresponds to a temperature difference between the wall and the bulk fluid of about 1 °C, and thus strong buoyancy induced effects are expected to be minimal. Temperature differences beyond about 100 °C are rare in practice, thus the applicability of the present equation is expected to cover most practical cases of buoyancy affected pressure drop in aiding flow in a vertical tube.

6. Conclusions

An equation has been developed to predict the pressure drop as a function of flow rate in aiding, mixed convection flows in a vertical tube for the laminar regime, as defined by heat transfer. This equation was developed using a core-shell model coupled with the momentum integral boundary layer approach. A shear rate modification was used to form the comparison of pressure drops, and a semi-empirical approach was used to determine constants that could otherwise not be obtained by analytical solution. This equation will be useful in design for most practical cases involving aiding flow in the laminar region, except for situations where the curvature affects the boundary layer (small diameter tubes). The equation is expected to be valid for Grashof numbers based on diameter of up to about 10^8 and Reynolds number up to about 11,000 or to where the flow becomes clearly turbulent.

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